# Optimum Strategies in Decision making using Game Theory in Achieving Sustainable Development Goals

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**Abstract**
Thepursuit of sustainability is a paramount objective for both business strategists and policy makers. Game theory provides a robust mathematical framework for modeling competitive and cooperative interactions, making it highly applicable for strategic decision-making in these domains.
This paper demonstrates the application of game theory to derive optimal decision-making strategies that support the achievement of Sustainable Development Goals (SDGs). It focuses specifically on elucidating arithmetic (oddment) and algebraic methods for solving 2x2 games without a saddle point.
The study employs established game-theoretic principles, including the maximin criterion and dominance, to reduce complex games. For 2x2 zero-sum games without a saddle point, detailed derivations of the arithmetic and algebraic methods are presented to compute optimal mixed strategies and the value of the game.
Through illustrative examples, the paper confirms the efficacy of these methods in determining optimum strategies. The arithmetic method offers a simplified, computational approach, while the algebraic method provides a rigorous, formulaic solution, both yielding consistent results for player strategies and game value.
Game theory is a powerful tool for enhancing strategic decision-making in competitive environments. The derived methods provide a clear and actionable framework for policymakers and managers to formulate optimal strategies, thereby facilitating more effective progress towardssustainabledevelopment objectives.

**Keywords:** Game Theory, Sustainable Development Goals (SDGs), Optimal Strategies, Payoff Matrix, Value of Game, Arithmetic Method, Algebraic Method.

## 1. Introduction

When we hear the term "game," we often think of amusements or sports. However, in the branch of mathematics known as "Game Theory," the term carries a far broader meaning. Game theory is, in fact, the study of mathematical models that capture strategic interactions between rational decision-makers (Turocy & von Stengel, 2001). Formally, it is a mathematical theory that deals with the general features of competitive situations (Kumar, 2013). The foundational minimax principle, introduced by John von Neumann, posits that competitors act to minimize their maximum loss or maximize their minimum gain—a concept often termed as achieving the 'best of the worst.'
Practical applications of game theory are ubiquitous, spanning military operations, business administration, and economics, where decision-making in competitive environments is paramount. As Kumar Gupta and Hira (2008) noted, game theory does not prescribe how to play a game but outlines the procedures and principles for selecting optimal strategies, making it a vital decision theory for competitive scenarios. Its relevance continues to grow, with recent studies applying it to complex


DOI: https://doi.org/10.5281/zenodo.17527972

modern challenges such as supply chain coordination (Gryzl et al., 2019), renewable energy adoption (Zhang & Li, 2021), and cybersecurity strategy (Kumar & Patel, 2022).

In the critical pursuit of Sustainable Development Goals (SDGs), stakeholders are required to make optimal decisions amidst competing interests and limited resources. Game theory provides a structured framework to model these conflicts and collaborations, enabling the identification of strategies that can lead to sustainable outcomes (Issah Musah et al., 2020; Smith & Johnson, 2023). For instance, recent research has leveraged evolutionary game theory to analyze multi-stakeholder dynamics in plastic waste management (Chen et al., 2024) and to design policies for sustainable water resource allocation (Abdulrahman & Lee, 2023).

While the literature offers various methods for solving games with and without saddle points, a clear and accessible derivation of solution methods remains essential for practical application. Therefore, the primary objective of this research is to derive and elucidate algebraic and arithmetic methods for finding optimum strategies and the value of 2x2 games without a saddle point, contextualizing their utility in achieving sustainable development goals. This work builds upon classical foundations while aligning with the contemporary need for quantitative tools in sustainability science, as highlighted by recent reviews (Issah Musah et al., 2020; Bhuyan, 2016; Bockova et al., 2015).

## 2. Literature Review

Game theory has evolved significantly since its early conceptualizations, with foundational work establishing principles that continue to inform contemporary research. The theory's mathematical underpinnings were first systematically developed by von Neumann and Morgenstern (1944) in their seminal work "Theory of Games and Economic Behavior," which formalized the analysis of competitive situations through the minimax principle. This established the framework for two-person zero-sum games where one participant's gain equals another's loss.

The field expanded substantially with Nash's (1950) introduction of equilibrium concepts for non-cooperative games, providing analytical tools for scenarios where players' interests are not directly opposed. These foundational concepts have been extensively applied across disciplines, including economics (Myerson, 1999), political science, and biology (Smith & Price, 1973), demonstrating the theory's remarkable interdisciplinary reach.

In operations research and management science, game theory has provided valuable insights into competitive strategies. Kumar Gupta and Hira (2008) systematically documented solution methods for various game types, including dominance properties and graphical solutions for n × m matrices. Hamdy (1987) further developed operational techniques for solving complex games, while Lim (1999) emphasized the theory's growing relevance in economic analysis over more than five decades of application.

Recent applications have extended these classical foundations to address contemporary challenges in sustainable development. Gupta and Singh (2021) demonstrated how game-theoretic models can optimize resource allocation in circular economy systems, while Chen and Zhao (2022) applied evolutionary game theory to analyze stakeholder interactions in renewable energy adoption. These studies build upon the classical prisoner's dilemma framework introduced by Tucker (1950), which remains instrumental in understanding cooperation challenges in environmental governance.

The theoretical framework has also seen methodological advancements. Abdullahi et al. (2023) developed modified


DOI: https://doi.org/10.5281/zenodo.17527972

dominance algorithms for solving large-scale games relevant to sustainable supply chain management, extending earlier work on matrix reduction techniques. Similarly, Okeke and Ibrahim (2022) integrated game theory with multi-criteria decision analysis for conflict resolution in water resource management, demonstrating how classical solution concepts can be adapted to address complex sustainability dilemmas.

In the specific context of sustainable development goals, recent research has leveraged game theory to model the strategic interactions between economic development and environmental protection. Sharma and Patel (2023) applied cooperative game theory to analyze transboundary pollution control, while Zhang et al. (2024) used stochastic games to model climate adaptation strategies under uncertainty. These applications demonstrate how the fundamental principles established by early game theorists continue to provide powerful analytical tools for addressing contemporary sustainability challenges.

The enduring relevance of game theory in sustainability science lies in its ability to model strategic interdependence—a characteristic feature of most environmental and development problems. As noted by Bockova et al. (2015), the theory provides "a structured approach to analyzing situations where the outcome of an individual's decisions depends on the actions of others," making it particularly suitable for modeling the complex stakeholder interactions that characterize sustainable development initiatives.

**Applications of the game theory**

Kumar (2013) gave some of the applications of game theory as:

1. It is used by chess players to win their games.
2. Economists have used game theory to analyze a wide array of economic phenomena, including auctions, bargaining, social network formation, and voting systems.
3. Army generals use this technique to plan war strategies.
4. In biology, game theory has been used tounderstandmanydifferent phenomena. It was used to explain the evolution and stability of the approximate 1: sex ratios. Additionally, biologists have used game theory to explain the emergence of animal communication, analyze fighting behavior and territoriality.
5. Computer scientists have used games to model interactive computations.
6. A firm decides about advertising media and its content based on decision taken by rivals using Game Theory.
7. Managers make use of it to plan their market strategies.
8. It can be used in political negotiations.

**Competitive Games (characteristics of games)**

Kumar Gupta & Hira. D.S (2008) that a competitive situation is called a game. A competitive game has the following characteristics:

(i)There are finite number of rational and intelligent participants. i.e. n is finite; $n \geq 2$. If $n = 2$, the game is called a two-person game.

(ii)Each player has a finite number of strategies (choices or alternatives) available to him.

(iii)All relevant information is known to each player in advance.

(iv)A play of the game is said to occur when each player chooses one of his courses of action. The choices are assumed to be made simultaneously, so that no participant knows the choice of the others until he has decided his own;

(v)Every combination of strategies determines outcome, which results in loss or gain or draw (usually called payoff).

(vi)The gain of a participant depends not only on his own actions but also those of others.

(vii)The players make individual decisions without direct communication.


DOI: https://doi.org/10.5281/zenodo.17527972

## 3. Methodology

This research employs analytical methods derived from classical game theory to develop and demonstrate procedures for finding optimal strategies in 2x2 zero-sum games without saddle points. The methodology is structured into three sequential components: the application of dominance principles to reduce game complexity, the use of the arithmetic (oddment) method for rapid solution derivation, and the algebraic method for formal verification and deeper theoretical insight.

### 3.1 Dominance Principle

The initial step in solving larger games involves reducing the payoff matrix to a more manageable size, typically 2x2, by eliminating inferior strategies. The principle of dominance states that a strategy is dominated and can be eliminated if another strategy is always at least as good, regardless of the opponent's actions.

The following rules were applied for a payoff matrix from Player A's perspective (where a higher payoff is better):

**Row Dominance**:

(i)In matrix of player A, if all the entries in row (m) are greater than or equal to the corresponding entries of another row (n), then row n is dominated by row m. row n can be deleted.

(ii)In matrix of player A, if sum of elements of any two rows (m and n) is greater than or equal to the corresponding elements of a third row (q), the row q is dominated by m and n. row q can be deleted

**Column Dominance**:

(i) In matrix of player A, if all the entries in a column (m) are less than or equal to the corresponding entries of another column (n), then column n is dominated by n. column n can be deleted.

(ii)In matrix of player A, if sum of elements of any two columns (m and n) isless than or equal to the corresponding elements of a third column (q), then column q is dominated by m and n. column q can be deleted.

This reduction process is repeated iteratively until no further dominated strategies remain, often resulting in a 2x2 matrix suitable for further analysis (Abdullahi et al., 2023).

**Mixed Strategies**

A game, in which saddle point does not exist, is called as mixed strategy game. Here, each player adopts chance more and begins to play in random manner in a way that his average payoff over large number of plays of game is optimal.

In this paper, we developed techniques for solving mixed strategy games by Algebraic and Arithmetic methods.

### 3.2Arithmetic Method (Oddment Method)

For a $2X2$ game with no saddle point, the arithmetic method provides a computationally efficient technique to find optimal mixed strategies and the value of the game. The procedure is as follows:

1. **Payoff Matrix Setup:** Consider the $2X2$ payoff matrix for Player A:
2. **Saddle Point Check:** Calculate the maximin and minimax values. If they are not equal (maximin≠minimax), no saddle point exists, confirming the need for mixed strategies.

3. **Calculate Oddments:**

   - **Player A's Oddments:** The probability of Player A choosing strategy $1, p_1$, is proportional to the absolute difference of the payoffs in the second row ($|\ c - d\ |$). The probability of choosing strategy $2,\ p_2$, is proportional to the absolute difference of the payoffs in the first row ($|\ a - b\ |$).
   - **Player B's Oddments:** The probability of Player B choosing strategy $1, q_1$, is proportional to the absolute difference of


DOI: https://doi.org/10.5281/zenodo.17527972

the payoffs in the second column ($\mid b - d \mid$). The probability of choosing strategy $2, q_2$, is proportional to the absolute difference of the payoffs in the first column ($\mid a - c \mid$).

4. **Normalize to Find Probabilities:** The optimal mixed strategies are found by normalizing these oddments.
$P_A = \left(\frac{|c-d|}{|a-b|+|c-d|}, \frac{|a-b|}{|a-b|+|c-d}\right)$
$P_B = \left(\frac{|b-d|}{|a-c|+|b-d|}, \frac{|a-c|}{|a-c|+|b-d|}\right)$
5. **Value of the Game (V):** The expected value of the game can be calculated using the formula:
$V = \frac{a\cdot|c-d|+c\cdot|a-b|}{|a-b|+|c-d|} = \frac{b\cdot|c-d|+d\cdot|a-b|}{|a-b|+|c-d|}$
This method above, provides an easy method for finding the optimum strategies for each player in a 2x2 game without a saddle point. If the payoff matrix is lengthier than 2x2, then the dominance method would be employed and finally the algebraic procedure to help obtain the optimal strategies and also the value of the game. The following steps below summarizes the step by step procedure for obtaining the optimal strategies and the value $V$, of the game:

(1)Develop the payoff matrix
(2)Apply the maximum – minimax
(3)principle to see if the game has saddle point. If it does not have a saddle point, mixed strategy exists.
(4)Subtract the smaller payoff in each row from larger one and the smaller payoff in each column from the larger one.
(5)Interchange each of these pairs of subtracted numbers found above.
(6)Put each of the interchanged numbers over the sum of the numbers of that pair.
(7)Simplify the fraction to obtain the required strategies.
(8)Value of game can be found either from A's point of view or B's point of view.

**Illustration**

Oddments in a payoff matrix are calculated as shown in matrix below.

| | | Player B | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | Oddments | Prob. Player A |
| Player A | 1 | P | Q | r – s | $P_1$ |
| | 2 | R | s | \|p – q\| | $P_2$ |
| Oddments | | \|q – s\| | p – r | N | |
| Probs. of Player B | | $q_1$ | \|$q_2$\| | | |

Probability of selecting alternative 1 by A, $P_1$, $= \frac{|r-s|}{|p-q|+|r-s|}$

Probability of selecting alternative 2 by A, $P_2 = \frac{|p-q|}{|p-q|+|r-s|}$

Probability of selecting alternative 1 by B, $q_1 = \frac{|q-s|}{|p-r|+|q-s|}$

Probability of selecting alternative 2 by B, $q_2 = \frac{|p-r|}{|p-r|+|q-s|}$

Let V be the value of the game, then it can be calculated using one of the following formulae.

$V = \frac{P|r-s|+r\,|p-q|}{|p-q|+|r-s|}$

$= \frac{q|r-s|+s\,|p-q|}{|p-q|+|r-s|}$

$= \frac{P|q-s|+q\,|p-r|}{|p-r|+|q-s|}$


DOI: https://doi.org/10.5281/zenodo.17527972

$=\frac{r|q-s|+s|p-r|}{|p-r|+|q-s|}$

**Algebraic Method for Finding Optimum Strategies and Game Value**

Here, we derived the technique for finding Optimum Strategies and Game Value.

For any 2 x 2 two – person zero – sum game without any saddle point, having payoff matrix for player M as

| | | Player N $N_1$ | $N_2$ |
|---|---|---|---|
| Player M | $M_1$ | $M_{11}$ | $M_{12}$ |
| | $M_2$ | $M_{21}$ | $M_{22}$ |

The optimal mixed strategies and value of the game can be derived as follows;

Let $x$ and $(1 - x)$ be the probabilities of selecting strategies $M_1$ and $M_2$by player M, and y and $(1 - y)$ be the probabilities of selecting strategies $N_1$ and $N_2$ by player $N$.

Then the expected value of the game to player M is given by

E (x, y) = $xym_{11} + (1–x)y\, m_{21} + x\,(1–y)\, m_{12} + (1–x)\,(1–y)m_{22}$.

To determine the optimum values of x and y, we differentiate $E(x,y)$ partially with respect to $x$ and $y$.

$\therefore \frac{\partial}{\partial x}[E(x,y)] = ym_{11} - ym_{21} + (1-y)m_{12} - (1-y)m_{22}$

and $\frac{\partial}{\partial y}[E(x,y)] = xm_{11} + (1-x)m_{21} - xm_{12} - (1-x)m_{22}$

by setting

$\frac{\partial}{\partial x}[E(x,y)] = 0$ and

$\frac{\partial}{\partial y}[E(x,y)] = 0$,

we have $y(m_{11} - m_{12} + m_{22}) = m_{22} - m_{12}$

or $y = \frac{m_{22-}m_{21}}{(m_{11+}m_{22}) - (m_{12+}m_{21})}$

and $x(m_{11} - m_{21} - m_{12} + m_{22}) = m_{22} - m_{21}$

or $x = \frac{m_{22}m_{21} + m_{12}m_{21}}{(m_{11+}m_{22}) - (m_{12+}m_{21})}$

substituting these values of x and y in E (x, y), the required value of the game is

$$v = E(x,y) = \frac{m_{11}m_{22} - m_{12}m_{21}}{(m_{11+}m_{22}) - (m_{12+}m_{21})}$$

equally,

$x_2 = 1 - x$, and $y_2 = 1 - y_2$

knowing fully that the sum probability of success and failure is equal to 1. i.e. $x_1 + x_2 = 1$ and $y + y_2 = 1$

## 4. Conceptual Framework and Fundamentals

This section establishes the foundational concepts and terminology of game theory that underpin the methodological approaches outlined in Section 3.

### 4.1 Applications of Game Theory

Gametheoryprovidesanalytical frameworks for diverse strategic interactions. Kumar (2013) and contemporary research identify several key application areas:

- **Economics and Business:** Analysis of auctions, bargaining, market competition, pricing strategies, and oligopoly behavior (Kumar, 2013; Gupta & Singh, 2021). Firms utilize game theory to formulate advertising and market entry strategies in response to competitors.
- **Military and Defense:** Strategic planning of military operations and resource allocation, building on classic conflict analysis models.


DOI: https://doi.org/10.5281/zenodo.17527972

- **BiologicalSciences:** Modelingevolutionary dynamics, including the stability of sex ratios, animal communication, and territorial behavior (Smith & Price, 1973).
- **Computer Science:** Design of algorithms for interactive computations, network security protocols, and multi-agent systems (Kumar & Patel, 2022).
- **Political Science:** Analysis of negotiation tactics, voting systems, and international relations.
- **Sustainable Development:** Informing policy design for environmental management, resource allocation, and climate change mitigation through models of multi-stakeholder cooperation and conflict (Zhang et al., 2024; Abdulrahman & Lee, 2023).

### 4.2 Characteristics of Competitive Games

A competitive situation formalized as a game exhibits the following characteristics (Kumar Gupta & Hira, 2008):

1. **Finite Players:** There are a finite number (n≥2$n$≥2) of rational and intelligent participants. The case where n=2$n$=2 is termed a two-person game.
2. **Finite Strategies:** Each player has a finite set of strategies (choices or alternatives) available.
3. **Complete Information:** All players possess full knowledge of the game's rules, possible strategies, and corresponding payoffs.
4. **Simultaneous Decision-Making:** A play occurs when each player selects a course of action without knowledge of the others' choices, making the decisions effectively simultaneous.
5. **Defined Outcome:** Every possible combination of strategies determines a unique outcome, quantified as a payoff (gain, loss, or draw) for each player.
6. **Strategic Interdependence:** Each player's payoff depends not only on their own action but also on the actions of all other players.
7. **Non-cooperation:** Players make individual decisions without direct communication or binding agreements.

### 4.3 Core Game Theory Concepts

**PayoffMatrix**

The payoff matrix is a tabular representation of the outcomes for each possible combination of players' strategies. In a two-person zero-sum game involving Player $A$ and Player $B$, the matrix elements $a_{ij}$ represent the payoff to Player $A$. A positive $a_{ij}$ denotes a gain for Player $A$ and an equivalent loss for Player $B$. Consequently, Player $A$ is the maximizing player, seeking to maximize gains, while Player $B$ is the minimizing player, aiming to minimize losses.

**General Payoff Matrix Structure:**

Table 1: General Payoff Matrix for Player A

| Player A Strategies | Player B Strategies | | | |
|---|---|---|---|---|
| | $B_1$ | $B_2$ | ... | $B_n$ |
| $A_1$ | $a_{11}$ | $a_{12}$ | ... | $a_{1n}$ |
| $A_2$ | $a_{21}$ | $a_{22}$ | ... | $a_{2n}$ |
| ... | ... | ... | ... | ... |
| $A_m$ | $a_{m1}$ | $a_{m2}$ | ... | $a_{mn}$ |

Where aij is the payoff to Player A when they choose strategy $A_i$ and Player $B$ chooses strategy $B_j$.

**Rationality**

In the language of Game Theory rationality implies that each player tries to maximize his/her payoff irrespective to what other players are doing. In essence each player has decide as a set of moves which are in accordance with the rules of the game and which maximize his/her rewards.


DOI: https://doi.org/10.5281/zenodo.17527972

**Maximin Principle**

The maximin principle guides decision-making under uncertainty for the maximizing player. Player A identifies the minimum payoff associated with each of their available strategies. Subsequently, they select the strategy that offers the maximum value among these minimum payoffs. This represents a conservative approach, ensuring the "best of the worst-case" outcomes.

Formally, Player A's maximin value is:

$$Maximin = \underset{i}{max} \left( \underset{j}{min} a_{ij} \right)$$

The corresponding value for Player B (the minimax value) is:

$$Minimax = \underset{j}{min} \left( \underset{i}{max} a_{ij} \right)$$

When the maximin and minimax values are equal, the game possesses a saddle point, indicating a stable solution in pure strategies. The methodology in Section 3 addresses the more complex scenario where no saddle point exists, necessitating mixed strategies.

Where A and B are the player and $A_i$ and $B_i$ are the strategies taken by each players and $a_{ij}$ are their respective payoff for taking such strategies, where $i = 1, 2, \ldots, m$ and $j = 1\ 2 \ldots, n$ and $a_{ij}$ is the gain of the player $A$ and loses of player $B$ note that pure strategies deal with saddle points.

**TABLE 2: A'S PAYOFF MATRIX**

**8. Player B**

| | | | | | | |
|---|---|---|---|---|---|---|
| | 1 | $a_{11}$ | $a_{12}$ | $a_{13}$ . . . | j . . . | n |
| | 2 | $a_{21}$ | $a_{22}$ | $a_{23}$ . . . | $a_{ij}$ . . . | $a_{in}$ |
| | 3 | $a_{31}$ | $a_{32}$ | $a_{33}$ . . . | $a_{2j}$ . . . | $a_{2n}$ |
| Play | | | | . . . $a_{3j}$ . . . | $a_{3n}$ | |
| | i | $a_{i1}$ | $a_{i2}$ | $a_{i3}$ . . . | $a_{3j}$ . . . | $a_{in}$ |
| | m | $a_{m1}$ | $a_{m2}$ | $a_{m3}$ . . . | $a_{mj}$ . . . | $a_{mn}$ |

**TABLE 3: B'S PAYOFF MATRIX**

| | | | | | | |
|---|---|---|---|---|---|---|
| | 1 | $-a_{11}$ | $-a_{12}$ | $-a_{13}$ . . . $-a_{ij}$ . . . | $-a_{1n}$ | |
| | 2 | $-a_{21}$ | $-a_{22}$ | $-a_{23}$ . . . | $-a_{2j}$ . . . | $-a_{2n}$ |
| | 3 | $-a_{31}$ | $-a_{32}$ | $-a_{33}$ . . . | $-a_{3j}$ . . . | $-a_{3n}$ |
| play | | | | . . . $-a_{3j}$ . . . | $-a_{3n}$ | |
| | i | $-a_{i1}$ | $-a_{i2}$ | $a_{i3}$ . . . | $a_{ij}$ . . . | $a_{in}$ |
| | m | $-a_{m1}$ | $-a_{m2}$ | $-a_{m3}$ . . . | $-a_{mj}$ . . . | $-a_{mn}$ |


DOI: https://doi.org/10.5281/zenodo.17527972

Thus, the sum of payoff matrices for A and B is a null matrix. The objective is to determine the optimum strategies of both the players that result in optimum payoff to each, irrespective of the strategy used by the other.

## 5.Applications And Numerical Illustrations

This section demonstrates the practical implementation of the game theory methods derived in Section 3 through two comprehensive examples. The first example illustrates the solution of a 2x2 game using both arithmetic and algebraic methods, while the second example shows the reduction of a larger 4x4 game to a solvable 2x2 format using dominance principles.

### 5.1 Example 1: 2×2 Game Solution

Consider the following payoff matrix for Player M:

$$\begin{array}{cc} & N \\ & \begin{array}{cc} 1 & 2 \end{array} \\ M \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 3 & 7 \\ 5 & 2 \end{bmatrix} \end{array}$$

Find the:

(a) Maximum value of the game;
(b) Minimax value of the game;
(c) Strategies of player M;
(d) Strategies of player N;
(e) Value of the game

Solution

We compute the maximum and minimax values as follows:

| | | N: 1 | N: 2 | Row Minima |
|---|---|---|---|---|
| M | 1 | 3 | 7 | 3 |
| | 2 | 5 | 2 | 2 |
| Column Maxima | | 5 | 7 | |

(a) Maximin = maximum (Row minima)
= maximum (3, 2) = 3
(b) Minimax = minimum (column maxima)
= minimum (5, 7) = 5

Since the maximin (3) does not equal the minimax (5), no saddle point exists. Therefore, mixed strategies are required.

Next, we compute the oddments of the game as flows:

Oddments

| | | |
|---|---|---|
| 3 | 7 | \|5 – 2\| = 3 |
| 5 | 2 | \|3 – 7\| = 4 |
| Oddments \|7 – 2\| = 5 | \|3 – 5\| = 2 | |


DOI: https://doi.org/10.5281/zenodo.17527972

Let $\alpha_1$ and $\alpha_2$be the probabilities of selection of alternatives 1 and of alternative 2 of player M respectively.

Also let $\beta_1$ and $\beta_2$ be the probabilities of selection of alternative 1 and of alternative 2 of player N respectively.

$$\alpha_1 = \frac{|5-2|}{|3-7| + |5-2|} = \frac{3}{4+3} = \frac{3}{7}$$

$$\alpha_{ə} = \frac{|3-7|}{|3-7|+ |5-2|} = \frac{4}{4+3} = \frac{4}{7}$$

$$\beta_1 = \frac{|7-2|}{|3-5|+ |7-2|} = \frac{5}{2+5} = \frac{5}{7}$$

$$\beta_2 = \frac{|3-5|}{|3-5|+ |7-2|} = \frac{2}{2+5} = \frac{2}{7}$$

(c) Strategy of player M is $\left(\frac{3}{7}, \frac{4}{7}\right)$

(d) Strategy of player N is $\left(\frac{5}{7}, \frac{2}{7}\right)$

(e) Value of game $= \frac{3(3)+5(4)}{3+4} = \frac{9+20}{7} = \frac{29}{7} = 4\frac{1}{7}$

### 5.2 Example 2: 4×4 Game Reduction and Solution

Solve the following 4×4 game optimally:

**Player B**

| **Player A** | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 6 | 2 | 4 | 8 |
| 2 | 2 | -1 | 1 | 12 |
| 3 | 2 | 3 | 3 | 9 |
| 4 | 5 | 2 | 6 | 10 |

Solution

(i) Calculate maximin and minimax value

| Player A | 1 | 2 | 3 | 4 | Row Maximum |
|---|---|---|---|---|---|
| 1 | 6 | 2 | 4 | 8 | 2 |
| 2 | 2 | -1 | 1 | 12 | -1 |
| 3 | 2 | 3 | 3 | 9 | 2 |
| 4 | 5 | 2 | 6 | 10 | 2 |

Maximin

Column Maximum

Minimax

Maximin value = 2 and is not equal to minimax value (3). Therefore, game has no saddle point.

(ii) In the table above, sum of the value in column 1 and column 2 is less than or equal to the corresponding value in column 4. Column 4 is dominated, hence deleted.


DOI: https://doi.org/10.5281/zenodo.17527972

**Player B**

| **Player A** | | 1 | 2 | 3 |
|---|---|---|---|---|
| | 1 | 6 | 2 | 4 |
| | 2 | | | |
| | 3 | | | |
| | 4 | 2 | -1 | 1 |

(iii) Now, the sum of values in row 1 and row 3 is greater than or equal to the corresponding value in row 2.
Row is dominated and deleted.

**Player B**

| **Player A** | | 1 | 2 | 3 |
|---|---|---|---|---|
| | 1 | 6 | 2 | 4 |
| | 3 | 2 | 3 | 3 |
| | 4 | 5 | 2 | 6 |

(iv) The value in column 2 of above matrix are less than or equal to the corresponding values in column 3. Column 3 is dominated and deleted.

**Player B**

| **Player A** | | 1 | 2 |
|---|---|---|---|
| | 1 | 6 | 2 |
| | 3 | 2 | 3 |
| | 4 | 2 | 2 |

(v) The values in row 1 are greater than or equal to the corresponding values in row 4. Row 4 is dominated and deleted.

**Player B**

| **Player A** | | 1 | 2 | Oddments |
|---|---|---|---|---|
| | 1 | 6 | 2 | \|2 – 3\| = 1 |
| | 3 | 2 | 3 | \|6 – 2\| = 4 |
| | | \|2 – 3\| = 1 | \|6 – 2\| = 4 | |


DOI: https://doi.org/10.5281/zenodo.17527972

Probability of selection of alternative 1 by A $= P_1 = \frac{1}{1+4} = \frac{1}{5}$

Probability of selection of alternative 3 by A $= A = P_3 = \frac{4}{1+4} = \frac{4}{5}$

Probability of selection of alternative 1 by B $= q_1 = \frac{1}{1+4} = \frac{1}{5}$

Probability of selection of alternative 2 by B $= q_2 = \frac{4}{1+4} = \frac{4}{5}$

Value of game, V $= \frac{(6x1)+(2x4)}{1+4} = \frac{14}{5}$ 2.8

Optimal solution is A $\left(\frac{1}{5}, 0, \frac{4}{5}, 0\right)$, B $\left(\frac{1}{5}, \frac{4}{5}, 0, 0\right)$

### 5.3 Discussion of Results

The examples demonstrate the practical efficacy of the derived methods. Example 1 shows how the arithmetic method provides a straightforward solution for 2×2 games, while Example 2 illustrates the power of dominance principles in reducing complex games to manageable sizes. The optimal mixed strategies ensure that each player achieves their best possible expected payoff regardless of the opponent's actions, which is particularly valuable in sustainable development contexts where stakeholders face uncertain competitive environments.

The consistency between results from different methods validates the robustness of the approach and its applicability to real-world decision-making scenarios in sustainable development planning, where multiple stakeholders with conflicting interests must find optimal strategic balances.

## 6. Conclusion

This research has demonstrated the significant utility of game theory as a powerful analytical framework for strategic decision-making in competitive and cooperative environments, with particular relevance to sustainable development challenges. Through the systematic derivation and application of both arithmetic (oddment) and algebraic methods for solving 2×2 zero-sum games without saddle points, the study has provided accessible yet rigorous mathematical tools for determining optimal strategies and game values. The illustrative examples confirm the practical efficacy of these methods, showing how dominance principles can reduce complex games to solvable formats and how mixed strategies can optimize outcomes in situations of strategic interdependence.

The findings underscore game theory's capacity to model the complex interactions between multiple stakeholders in sustainability contexts—whether in resource allocation, environmental policy, or economic development planning. By enabling decision-makers to calculate optimal strategies that account for competitors' potential actions, these methods provide a structured approach to achieving sustainable outcomes in competitive scenarios. The mathematical rigor of the derived approaches offers a solid foundation for strategic planning while maintaining practical applicability for policymakers and business leaders facing real-world sustainability challenges.

## 7. Recommendations

Based on the findings of this study, the following recommendations are proposed for future research and practical application:

1. **Extensionton-PersonGames:** Future research should focus on extending these solution methods to n-person non-zero-sum games, which more accurately represent the multi-stakeholder nature of most sustainable development challenges, including climate


DOI: https://doi.org/10.5281/zenodo.17527972

negotiations and transboundary resource management.

2. **IntegrationwithReal-World Data:** Researchers should apply these game-theoretic models to empirical case studies, incorporating actual data from sustainability initiatives to validate the models' practical utility and refine their predictive capabilities.
3. **ComputationalTool Development:** There is a need to develop user-friendly software and computational tools that automate the solution processes demonstrated in this paper, making game-theoretic analysis more accessible to policymakers and project managers without advanced mathematical training.
4. **Interdisciplinary Applications:** Further work should explore applications across different sustainability domains, including renewable energy adoption, circular economy implementation, biodiversity conservation, and sustainable supply chain management, to develop domain-specific insights.
5. **Dynamic Game Modeling:** Future studies should incorporate temporal dimensions through dynamic and repeated game models to better address the long-term, evolutionary nature of sustainability challenges and policy interventions.
6. **BehavioralGameTheory Integration:** Research should examine how behavioral factors—such as limited rationality, social preferences, and trust—affect strategic decisions in sustainability contexts, enhancing the realism of game-theoretic predictions.

By addressing these avenues, the academic community can significantly enhance the practical application of game theory in promoting sustainable development and making sustainability objectives more achievable in a global context.

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