Mathematical Modelling and Application to Climate Change and Environmental Issues

Felix Eli Wang; Gau Patrick Damulak; Manjel Danladi
Department of Mathematics and Statistics, Plateau State Polytechnic,
Barkin-Ladi, Nigeria

Abstract
Climatechangeandenvironmentaldegradation pose pressing global challenges, necessitating innovative solutions to mitigate their impacts.
Mathematical modelling offers a powerful tool for understanding complex climate and environmental systems, enabling policy makers to make informed decisions.
This paper explores the application of mathematical modelling to climate change and environmental issues, focusing on temperature modelling, transportation modelling, and game theoretic approaches, recent advances, challenges and future directions are discussed, highlighting the critical role of mathematical modelling in informingclimatepolicy and environmental decision-making

Keywords: MathematicalModelling, Environmentalissues,Temperature modelling, game theory

1.Introduction

Climatechangeandenvironmentaldegradation have emerged as two of the most pressing global challenges of the 21st Century. Rising global temperatures, sea – level rise and extreme weather events have severe impacts on ecosystems, human health and economic stability.
The urgent need for effective climate actionandenvironmentalsuitabilitynecessitates innovative solutions, including the application of mathematical modelling.
Mathematical modelling offers a powerful tool for understanding complex climate and environmental systems, enabling policy makers to:

1. Analyze climate dynamics and predict future changes
2. Evaluate the effectiveness of climate policies and strategies
3. Identify optimal solutions for emission reduction and environmental sustainability
4. Inform decision – making process with data – driven insights

   The paper review recent advances in mathematical modelling applied to climate change and environmental issues, focusing on:

1.Temperature modelling: Global Climate Models (GCMs) and Regional Climates Models (RCMs)
ii.TransportationModelling:Optimalrouting and network flow models
iii.Game–theoreticapproaches:
Cooperative and non – cooperative game theory
By exploring the intersection of mathematical modelling, climate change, and environmental issues, this paper aims to contribute to the development of effective solutions for mitigating these pressing global challenges.

2.Literature Review

2.1Temperature Modelling

Global Climate Models (GCMs) and Earth System Models (ESMs) remain the principal tools for physically based climate simulation. CMIP6 (the Coupled Model Intercomparison Project phase 6) provides a standardized set of model experiments and forcings, paired with Shared Socioeconomic Pathways (SSPs) for future

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scenario analysis; these data underpin the Working Group I and synthesis reports of the IPCC AR6 (Eyring et al., 2016; WCRP CMIP6, 2022; IPCC, 2023). Regional Climate Models (RCMs) perform dynamical downscaling to give high-resolution projections required by impact assessments and local adaptation planning (Eyring et al., 2016; WCRP CMIP6, 2022).

Recent applications using CMIP6 datasets include regional drought and hydrological risk assessments (Ngwenya & Simatele, 2024), showing how scenario-based projections can identify future hotspots of climate risk. Ensemble model approaches and bias correction methods improve robustness for impact studies (IPCC, 2023).

2.2Transportation Modelling

Kumar Gupta and D. S. Hira (2012) gave the procedures for the formulation and finding the solution of transportation model with numerous examples

Nagurney et al (2017) optimized supply chain network for emission reduction

Wang et al (2018), developed a transportation modelling framework for urban planning

Vivek Kumar (2012) gave the procedures for using all the methods for solving transportation problems which includes: North West Corner Cell Method, Vogel's Approximation Method, Stepping Stone Method, the Modified Distribution Method and also how to test for the optimality of the solution.

2.3.Game Theoretic Approaches

Transportation and logistics are major emission sources. Optimization approaches (linear and nonlinear programming, network flow models) are used to minimizecostsandemissionssimultaneously, and to design resilient low-carbon transport systems (Nagurney et al., 2017; Wang et al., 2018). Recent work couples supply-chain network models with sustainability metrics and competition (Nagurney et al., 2017; Nagurney, 2025 summary), and AI techniques are increasingly integrated for dynamic routing and fleet optimisation under uncertainty.

3.Mathematical Models

3.1.Temperature Modelling

3.1.1. Global Climate Models (GCMs):

Simulate temperature responses to greenhouse gas emission.

$$\Delta T(t) = \lambda(C(t) - C_0) + \alpha T(t-1) + \varepsilon(t) \quad (1)$$

Where

$\Delta T(t)$ = temperature change at time t,

λ = Climate Sensitivity

$C(t)$ = CO_2 Concentration

C_0 = Pre-industrial CO_2 concentration, typically measured in parts per million (ppm)

Pre-industrial CO_2 concentration refers to the level of CO_2 in the atmosphere before the industrial Revolution. Such a representation is useful for policy analysis and as a reduced–form emulator of more complex GCM behavior (IPCC, 2023; Eyring et al., 2016). This value serve as a baseline for comparing current and future CO_2 concentrations

α = autocorrelation coefficient, and

$\varepsilon(t)$ = error term

3.1.2.Regional Climate Models (RCMs)

These are numerical models that simulate climate conditions over specific region, such as continents, countries, or even cities.

They are designed to provide detailed, high – resolution climate information for areas where global climate models (GCMs) may not be sufficient.

Characteristics of RCMs:

1. Higher Resolution: 10 – 50 km grid spacing, compared to GCMs (100 – 500km)
2. Regional Focus: Simulate Climate conditions for Specific regions or domains

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3. Nested within GCMs: RCMs use GCM output as boundary conditions
4. Improved representation of regional processes: topography, coastlines, land – use changes

Advantages of RCMS:

1. Better representation of regional climates variability
2. Improved simulation of extreme weather events
3. Enhanced resolution for impact assessments (e.g. agriculture, water resources)
4. Ability to incorporate regional – scale processes. (e.g. urban heat islands)

Applications of RCMS

1. Climates change impact assessments
2. Weather forecasting and prediction
3. Hydrological modelling
4. Agricultural planning and management
5. Urban planning and climate resilience

Regional Climate Models (RCMS) Derivation

Atmospheric Dynamics

1. Navier – Stokes Equations:

$$\frac{\partial u}{\partial t} + u\nabla u = -\frac{1}{\rho}\nabla p + v\nabla^2 u$$

$$\frac{\partial v}{\partial t} + v\nabla v = -\frac{1}{\rho}\nabla p + v\nabla^2 v$$

$$\frac{\partial w}{\partial t} + w\nabla w = -\frac{1}{\rho}\nabla p + v\nabla^2 w$$

Where:
u, v, w = wind components
ρ = Air density
p = pressure
v = viscosity

Thermodynamic Equation

$$\frac{\partial T}{\partial t} + u\nabla T = \frac{Q}{CP} - \frac{p}{\rho}\nabla T$$

Where:
T = Temperature
Q = Heat source/ Sink
CP = Specific Heat Capacity

Hydrology

Water Balance Equation

$$\frac{\partial S}{\partial t} = P - ET - Q$$

Where:
S = Soil moisture
P = Precipitation
ET = Evapotranspiration
Q = Runoff

Land – Surface Process:

Energy Balance Equation:

$$Rnet = Qh + Qe + Qg$$

Where:
$Rnet$ = Net radiation
Qh = Sensible Heat flux
Qe = latent heat flux
Qg = ground heat flux

Boundary Condition

Lateral Boundary Conditions:

$$u(x,y,t) = u_{GCM}(x,y,t)$$
$$v(x,y,t) = v_{GCM}(x,y,t)$$

Where
u, v = wind components at lateral boundaries
u_{GCM}, v_{GCM}= Wind components from Global Climate Model (GCM)

These equations form the core regional climate models, simulating atmospheric dynamics, hydrology, Land – surface processes and boundary conditions
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3.2.Transportation Model Applications

Transportation Model is used in

1. Route Optimization: Optimize routes forlogisticsandtransportation companies to reduce emissions
2. Traffic Management: Improve traffic management and reduce congestion using transportation modelling
3. Public Transportation Planning: Plan efficient public transportation systems
4. Electric Vehicle Infrastructure: Optimize electric vehicle charging infrastructure placement
5. Supply Chain Management: Optimize supply chain networks for emission reduction

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3.2.1.Formulation of Transportation Model

Consider a transportation problem with 'm' sources and 'n' destinations.

Let s_i be the supply at source i

Let d_j be the demand at destination j

Let c_{ij} be the cost of transportation of one units of products from i^{th} source to j^{th} destination

Let x_{ij} be the number of units to be transported from i^{th} source to j^{th} destination

Mathematically, transportation problem can be expressed as linear programming problem as follows:

Minimize total cost, $z = \sum_{i=0}^{m} \sum_{i=0}^{n} c_{ij} x_{ij}$

Subject to constraints $\sum_{i=0}^{n} x_{ij} = s_i (i = 1,2, \dots, m)$

$\sum_{i=0}^{m} x_{ij} = d_j (i = 1,2, \dots, m)$ and $x_{ij} \geq 0$

If $\sum_{i=0}^{m} s_i = \sum_{i=0}^{n} d_j$, the given transportation problem is balanced. Here the total supply is equal to the total demand and if $\sum_{i=0}^{m} s_i \neq \sum_{i=0}^{n} d_j$, the given problem is unbalanced

3.3.Game–TheoreticApproached (Applications)

1. InternationalClimateAgreements:Analyze the effectiveness of international climate agreements
2. Emissions Trading: Design and Evaluate Emission Trading Systems
3. Climate Change Mitigation Strategies: Evaluate climate mitigation strategies
4. Cooperative Game Theory: Analyze cooperative behaviour among countries and stakeholders
5. Non-Cooperative Game Theory: Analyze non – cooperative behaviours among countries and stakeholders.

3.3.1.Game–Theoretic Approach Model: International Climate Agreements

Goal: Evaluate the effectiveness of the Paris agreement where Paris agreement is an international accord aimed at mitigating climate change by limiting global warming to well below 2^oc (3.6^oF) above pre – industrial levels and pursuing efforts to limit to 1.5^oc (2.7^oF).

Model:NashEquilibrum:

$Max \sum_i u_i (x_i, x - i)$

Subject to:

$\sum_i x_i \leq 1$ (global emission cap)

$u_i(x_i, x - 1) = \beta_i(1 - x_i) + \gamma_i(x - 1)$

Where

u_i= Utility function for country i

x_i= emission reduction by country i

$(x - 1)$ =emission reduction by other countries

β_i, γ_i = coefficients

4. Application of the Models

4.1.TemperatureModelling Applications

4.1.1.Climate Policy Evaluation:

Evaluate the effectiveness of climate policies using temperature modelling:

$$\Delta T(t) = \lambda(C(t) - CO) + \alpha T(t - 1) + \varepsilon(t)$$

Where

$\Delta T(t)$ = temperature change at time t,

λ = Climate Sensitivity parameter

$C(t)$ = CO_2 Concentration

CO = Pre-industrial CO_2 concentration

α = Temperature persistence parameter

$T(t - 1)$ = Pervious temperature

$\varepsilon(t)$ = error term

Example 4.1:

Evaluate the impact of a 5% reduction in CO_2 emission on global temperature

4.1.2.ClimateChangeImpact Assessment

Assess the impacts of climate change on ecosystems using temperature modelling:

$y = \beta_0 + \beta_1 T + \beta_2 T^2 + \varepsilon$

Where:

y = Impact Variable (e.g., crop yield)

T = Temperature

$\beta_0, \beta_1, \beta_2$ = Coefficients

ε = Error term

Example 4.2

Assess the impact of a 2^oc temperature increase on global food production

4.1.3.Weather Forecasting

Improve weather forecasting accuracy using temperature modelling:

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$$T(t+1) = \emptyset T(t) + \theta T(t-1) + \varepsilon(t+1)$$

Where:
$T(t+1)$ = Forecasted temperature at time t + 1
$\emptyset, \theta$ = Autoregressive parameters
$T(t), T(t-1)$ = Current and previous temperatures
$\varepsilon(t+1)$ = Error term

Example 4.3
Forecast temperature for the next 24 hours

Example 4.4
$C(t)$ = 420 ppm (Current CO_2 concentration)
CO = 280 ppm (Pre-industrial CO_2 concentration)
λ = 0.01 (Climate sensitivity parameter)
Then $\Delta T(t) = 0.01(420 - 280) + \alpha T(t-1) + \varepsilon(t)$
The difference between $C(t)$ and CO (140 ppm) represents the increase in CO_2 concentration since pre-industrial times, driving the temperature change.

4.2.TransportationModelling Applications

4.2.1.Route Optimization:

Optimize routes for logistics and transportation companies: Minimize $\sum_{i,j} c_{ij} x_{ij}$
Subject to $\sum_j x_{ij} = 1 \quad \forall\, i \in N$ (nodes)
$\sum_i x_{ij} = 1 \quad \forall\, j \in N$ (nodes)
$x_{ij} \geq 0 \quad \forall\, i,j \in A$
Where:
c_{ij} = Travel cost between nodes i and j
x_{ij} = binary variable (1 if arc ij is used, 0 otherwise)
N = Set of nodes
A = Set of arcs

Examples 4.4
Optimize routes for a logistics company with 100 vehicles.

4.2.2 .Traffic Management

Improve traffic management using transportation modelling:
Maximize $\sum_{i,j} v_{ij} x_{ij}$
s.t.$\sum_j x_{ij} \leq c_i \quad \forall\, i \in N$ (nodes)
$x_{ij} \geq 0 \quad \forall\, i,j \in A$ (arcs)
Where:
v_{ij} = travel volume between nodes i and j
c_i = capacity constraint at node i
x_{ij} = binary variable (1 if arc ij is used, 0 otherwise)
Example: Optimize traffic signal control to reduce congestion.

4.3.Game–TheoreticApproaches Applications

4.3.1.CooperativeGameTheory: Prisoner's Dilemma

The prisoners, John and Matthew, are arrested and interrogated separately by the police for a crime they committed together.
The payoffs:

	John stays silent	John Betrays Matthew
Matthew Stay Silent	**3 years each**	**John 1 Year, Matthew 10 years**
Matthew Betrays John	**John 10 years, Matthew 1 year**	**John 5 years, Matthew 5 years**

The Dilemma:
Both prisoners have two options:

1. Stay silent (Cooperate)
2. Betray the other (Defect)

Rational Analysis
From John's perspective:
If Matthew stays, John gets 3 years by staying silent or 1 year by betraying
If Matthew betrays, John gets 10 years by staying silent or 5 year by betraying
From Matthew's perspective:

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- If John stays silent, Matthew gets 3 years by staying silent or 1 year by betraying
- If John betrays, Matthew gets 10 years by staying silent or 5 years by betraying

Nash Equilibrium

The Nash Equilibrium occurs when both prisoners betray each other, resulting in:
John: 5 years
Matthew: 5 years
This is the worst outcome for both, but it's the rational choice because:

- Each prisoner has an incentive to betray, regardless of the other's action
- Cooperation (Staying silent) is not sustainable

Another Example on Prisoner's Dilemma:
Two prisoners, A and B, can cooperate (C) or defect D, Payoffs:

	A(C)	**A(D)**
B(C)	**3,1**	**0,5**
B(D)	**5,0**	**2,2**

4.3.2 Non – Cooperative Game Theory

Nash Equilibrium

Firm 1 and Firm 2 set prices (High or Low). Payoffs:

	Firm 2 (High)	**Firm 2 (Low)**
Firm 1 (High)	**5,5**	**0,10**
Firm 2 (Low)	**10,0**	**3,3**

4.3.3. International Climate Agreements:

Analyze the effectiveness of international climate agreements:

Nash Equilibrium

$$\max \sum_i u_i\,(x_i, x-i)$$

S.t.$\sum_i x_i \leq 1$ (global emission cap)

$u_i(x_i, x-i) = \beta_i(1-x_i) + \gamma_i x - i$

Where:

u_i = Utility function for country i

x_i = emission reduction by country i

$x - i$ = emission reduction by other countries

β_i, γ_i = Coefficients

Example: Evaluate the effective of the Paris agreement

4.3.4.Emission Trading

Design and evaluate the emission trading system:

$$\sum_i \overline{x_i}\,(x_i)$$

s.t.

$\sum_j x_i \leq c$ (emission cap)

$\overline{x_i}(x_i) = P_i x_i - c_i x_i^2$

Where:

$\overline{x_i}$ =profit function for form i

x_i = emission allowance for form i

p_i= price of emission allowance

c_i = marginal abatement cost.

5. Conclusion

This study explored the integration of mathematical modelling techniques to address climate change and environmental issues, focusing on temperature modelling, transportation modelling, and game-theoretic approaches.

The research demonstrated the effectiveness of these methods in:

1. Temperaturemodelling: predicting global temperature increases and regional climate patterns.
2. Transportation modelling: optimizing fuel efficiency, reducing emissions, and promoting sustainable transportation.
3. Game- theoretic approaches: analysing cooperation, competition, and decision – making among stakeholders.

Finally, by advancing temperature modelling, transportation modelling, and game – theoretic approaches, we can

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develop effective strategies to mitigate and adopt to the climate crisis.

6. Recommentations

1. Interdisciplinary collaboration among mathematicians, climate scientists, policymakers,andtransportation experts.
2. Increased investment in climate modelling and research.
3. Enhanced education and training programs for climate modelling professionals.

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